Pak J. Statist.
2004 Vol. 20(2) pp 245 – 261

A STATISTICAL APPROACH TO COMBINING ENVIRONMENTAL INDICES WITH AN APPLICATION TO AIR POLLUTION DATA FROM BANGKOK, THAILAND

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ABSTRACT

In this paper we concentrate on the air pollution data measured as carbon monoxide, nitrogen dioxide, sulfur dioxide and ozone from ten monitoring stations in Bangkok, Thailand and apply Multiple Criteria Decision Making (MCDM) method to compute an overall air pollution index for these stations and compare them. We also study robustness of these overall indices.

1. INTRODUCTION

Multiple Criteria Decision Making (MCDM) has recently been recognized as an efficient statistical method to combine component 'indices' arising from many 'sources' into a single overall meaningful index. Such an index can be effectively used to compare relevant 'facilities'. The basic premise is a data matrix $X = (x_{ij}) : K \times N$ where the rows represent facilities which need to be compared or ranked with respect to the element x_{ij}'s, the columns represent various sources of the elements x_{ij}'s and x_{ij}'s themselves represent some quantitative information about the facilities. In the context of environmental science, the x_{ij}'s may represent levels of pollutants, facilities represent the sources of the pollutants (e.g., chemical or nuclear facilities) and the columns represent different types of pollution. Since usually it is difficult to compare the facilities on a multiple scale, MCDM provides a statistical method to combine the elements in any row into a single value which can then be used to compare the rows on a linear scale.

In this paper we briefly review MCDM in Section 2 and apply this technique and some of its variations to the air pollution data from Bangkok, Thailand in Section 3. Some conclusions are drawn in Section 4.

2. MCDM AND ITS MODIFICATIONS

In this section we briefly describe the Multiple Criteria Decision Making procedure and some of its variations.

2.1 A Brief Description

MCDM is a procedure to integrate multiple indicators into a single meaningful and overall index by combining $(x_{i1},\ldots,x_{iN})$ for row i across all indicators $j = 1,2,\ldots,N$. We can define an Ideal Row as one with the smallest observed value for each column

$$\text{IDR} = (\min_i x_{i1},\ldots,\min_i x_{iN}) = (u_1,\ldots,u_N)$$

and a Negative-ideal Row (NIDR) as one with the largest observed value for each column

$$\text{NIDR} = (\max_i x_{i1},\ldots,\max_i x_{iN}) = (v_1,\ldots,v_N).$$

For any given row i , we now compute the distance of each row from Ideal row and from Negative Ideal row based on the L_2-norm by using the formulae :

$$L_2(i,\text{IDR}) = [\sum_{j=1}^{N} \frac{(x_{ij}-u_j)^2 w_j}{\sum_{i=1}^{K} x_{ij}^2}]^{1/2}$$

$$L_2(i,\text{NIDR}) = [\sum_{j=1}^{N} \frac{(x_{ij}-v_j)^2 w_j}{\sum_{i=1}^{K} x_{ij}^2}]^{1/2}$$

where $w_1, w_2,\ldots,w_N$ are suitably chosen nonnegative weights between 0 and 1. The denominator above plays the role of a 'norming' factor. An objective way to select the weights is to use Shannon's [4] entropy measure φ based on the proportion $p_{1j},\ldots,p_{Kj}$ for the jth column where

$$p_{ij} = x_{ij} / \sum_{i=1}^{K} x_{ij} .$$

For the jth column, ϕ_j is computed as

$$\phi_j = \sum_{i=1}^{K} p_{ij} \ln(p_{ij}) / [\ln(K)].$$

The quantity φ essentially provides a measure of closeness of the different proportions. The smaller the value of φ, the larger the variation among the proportions for classifying the rows. So we can select the weights as

$$w_j = (1-\phi_j)/[\sum_{j=1}^{N}(1-\phi_j)] \quad , \quad j=1,\ldots,N .$$

In addition to Shannon's entropy measure, we can also use the sample variance of these proportions, given by

$$s^2_{j/prop} = \sum_{j=1}^{N}(p_{ij}-\overline{p}_j)^2/(K-1) .$$

If $\overline{x}_j$ and s_j^2 denote the mean and variance of x_{ij} in the jth column, $s^2_{j/prop}$ is directly proportional to $s_j^2/\overline{x}_j^2$, which is the square of the sample coefficient of variation cv_j. Therefore we propose to use $w_j = cv_j$.

The various rows are now ranked based on an overall index I computed as

$$I_i = \frac{L_2(i, IDR)}{L_2(i, IDR) + L_2(i, NIDR)} , \quad i=1,\ldots,K .$$

In addition to L_2-norm we can also use the L_1-norm as a distance measure and rank the rows once again. L_1-norm distance is defined below and the denominator again is used as a 'norming' factor.

$$L_1(i, IDR) = \sum_{j=1}^{N}\frac{|x_{ij}-u_j|w_j}{\sum_{i=1}^{K}x_{ij}}$$

$$L_1(i, NIDR) = \sum_{j=1}^{N}\frac{|x_{ij}-v_j|w_j}{\sum_{i=1}^{K}x_{ij}} \quad .$$

2.2 Modifications of MCDM

Here we describe two modifications of MCDM (Sinha and Shah, 2002). Let $d_i=[d_{i1},d_{i2},\ldots,d_{iN}]$ which represents the row-vector of d_{ij}'s, distance of x_{ij} from $\min_i x_{i:j}$, $1\le i\le K, 1\le j\le N$, for i-th row involving N columns, and

$d_i^- = [d_{i1}^-, d_{i2}^-, \ldots, d_{iN}^-]$ which represents the row-vector of d_{ij}^-'s, distance of x_{ij} from $\max_i x_{i:j}$, $1 \le i \le K, 1 \le j \le N$, for i-th row involving N columns.

Modification I:

$$L_i(d, d^-) = \left[\sum_j w_j (d_{ij}^k / d_{ij}^{-k}) / \left[\sum_i x_{ij}^2 \right]^{k/2} \right]^{1/2} + \left[\sum_j{}' w_j R_j^k / \left[\sum_i x_{ij}^2 \right]^{k/2} \right]^{1/2}$$

where Σ refers to all j for which $d_{ij}^- > 0$ while Σ' refers to all j for which $d_{ij}^- = 0$ and R_j is a finite quantity of our choice subject to $R_j \ge \max [d_{ij} / d_{ij}^-]$ taken over all i for which $d_{ij}^- > 0$.

Modification II:

$$L_i(d, d^-) = \left[\sum_j (w_j d_{ij}^k) / \left[\sum_i x_{ij}^2 \right]^{k/2} \right]^{1/2} + \left[\sum_j{}' (w_j / d_j^{-k}) / \left[\sum_i x_{ij}^2 \right]^{k/2} \right]^{1/2}$$
$$+ \left[\sum_j{}'' (w_j / r_j^2) / \left[\sum_i x_{ij}^2 \right]^{k/2} \right]^{1/2}$$

where $r_j \ge \min \{d_{ij}^-\}$ being taken over all $d_{ij}^- > 0$ and Σ' refers to all j for which $d_{ij}^- > 0$ while Σ'' refers to all j for which $d_{ij}^- = 0$. In the above k is a positive number.

To check the robustness of various sets of ranks produced by different methods, we will compute Spearman's rank correlation (SRC) coefficient :

$$r = 1 - \frac{1}{K(K^2 - 1)} \sum_{i=1}^{K} \Delta_i^2$$

where $\Delta_i =$ difference between ranks. It is obvious that a large value of r signifies a good agreement.

2.3 Electre Method

Electre Method (Sinha and Shah, 2002) is used for comparing the status of two locations rather than ranking all of them together. We begin with the $K \times N$ data matrix X of observations and proceed as follows:

Step 1:

Transform $X = [X_1, X_2, ..., X_N]$ to $R = [R_1, R_2, ..., R_N]$ where $R_i = \frac{X_i}{\|X_i\|^2}$.

Step 2:

Transform R to $V = RW$ where $W = \mathrm{diag}[w_1, w_2, ..., w_N]$.

Step 3:

Construct two matrices **C** and **D**

where $$c_{ij} = \sum_{k: v_{ik} \le v_{jk}} w_k \quad \text{and} \quad d_{ij} = \frac{\max_{k: v_{ik} < v_{jk}} |v_{ik} - v_{jk}|}{\max_k |v_{ik} - v_{jk}|}.$$

Compute $$\bar{c} = \frac{\sum\sum_{i \ne j} c_{ij}}{K(K-1)} \quad \text{and} \quad \bar{d} = \frac{\sum\sum_{i \ne j} d_{ij}}{K(K-1)}.$$

Step 4:

Construct matrices **F** and **G** such that

$$f_{ij} = \begin{cases} 1 \; ; c_{ij} \le \bar{c} \\ 0 \; ; \text{otherwise} \end{cases} \quad \text{and} \quad g_{ij} = \begin{cases} 1 \; ; d_{ij} \le \bar{d} \\ 0 \; ; \text{otherwise} \end{cases}.$$

Step 5:

Define matrix **E** where $e_{ij} = f_{ij} \cdot g_{ij}$.

It should be noted that the weights w_i's are obtained as discussed before, and that $e_{ij} = 0$ means that row i is better than row j.

3. AN APPLICATION

In this section we apply the previously described MCDM method and its modifications to the air pollution data from Bangkok, Thailand. The main air pollutants in Bangkok are carbon monoxide (CO_2), nitrogen dioxide (NO_2) and sulfur dioxide (SO_2) which are released directly from motor vehicles. The photochemical reaction on the oxide of nitrogen is ozone (O_3) which is a secondary pollutant.

The data sets were provided by the Pollution Control Department of Thailand and were recorded by 10 monitoring stations in Bangkok during 1998 – 2001. The monitoring stations are as follows:

1. Ramkhamheang University
2. National Housing Authority
3. Huai Khwang
4. Nonsee Vitaya School
5. Singharatpitayakom School
6. Thonburi
7. Chokchai 4
8. Dindaeng
9. Meteorological Department
10. Ratburana.

The locations of 10 monitoring stations in Bangkok are shown in Figure 1.

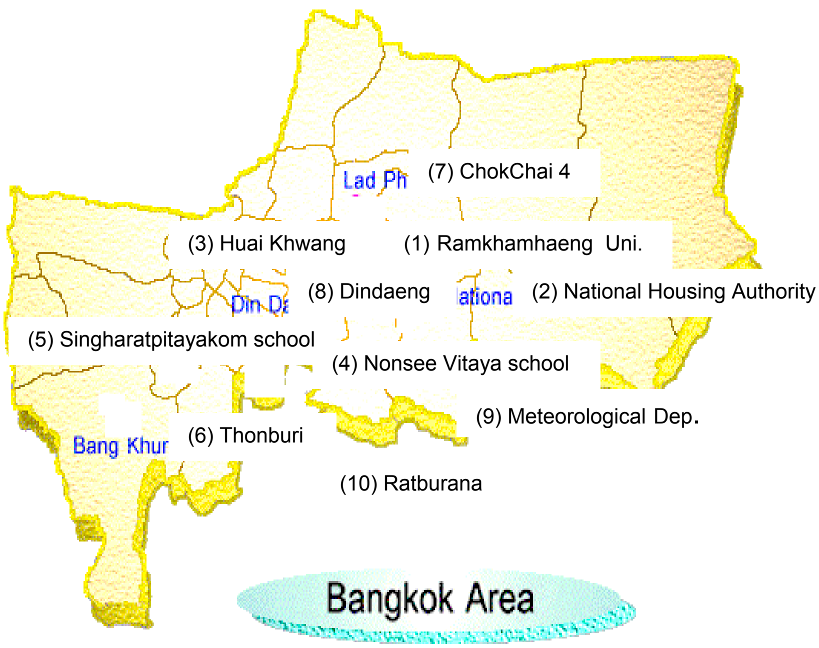

Figure 1 : Location of 10 monitoring stations in Bangkok area

At each station, the signals from the instruments were sampled every five seconds and hourly average values were calculated and stored. For our analysis, we have used the annual averages of each pollutant. The entire data set appears in a Technical Report (Lertprapai et al., 2003).

To apply the MCDM method, we use both the distance measures L_1 and L_2 as well as the two choices of weights based on phi and coefficient of variation (cv). We show below the results in four sets of the values of combined indices for each year. The final ranks of the rows are then based on the average index. We also compute the standard deviation to show the closeness of the four indices in a row. Tables 1 – 4 present all the results for years 1998 - 2001.

Table 1: Results of MCDM Method on Air Pollution Data in 1998.

Monitoring Station	L1		L2		Mean	SD	Rank
	W1	W2	W1	W2			
1) Ramkhamheang University	0.3574	0.3610	0.3891	0.3922	0.3749	0.0183	6
2) National Housing Authority	0.2983	0.3023	0.3327	0.3359	0.3173	0.0197	4
3) Huai Khwang	0.4423	0.4461	0.4390	0.4425	0.4425	0.0029	9
4) Nonsee Vitaya school	0.2934	0.2896	0.3271	0.3235	0.3084	0.0196	3
5) Singharatpitayakom school	0.3707	0.3685	0.3932	0.3937	0.3815	0.0138	7
6) Thonburi	0.4255	0.4293	0.4281	0.4316	0.4286	0.0025	8
7) Chokchai 4	0.3685	0.3665	0.3716	0.3698	0.3691	0.0021	5
8) Dindaeng	0.8054	0.7983	0.6909	0.6855	0.7450	0.0657	10
9) Meteorological Department	0.0387	0.0402	0.0492	0.0503	0.0446	0.0060	1
10) Ratburana	0.1217	0.1236	0.1828	0.1858	0.1535	0.0356	2

Table 2: Results of MCDM Method on Air Pollution Data in 1999.

Monitoring Station	L1		L2		Mean	SD	Rank
	W1	W2	W1	W2			
1) Ramkhamheang University	0.4414	0.4374	0.4683	0.4667	0.4534	0.0163	9
2) National Housing Authority	0.3281	0.3154	0.3709	0.3634	0.3445	0.0269	3
3) Huai Khwang	0.4271	0.4172	0.4287	0.4190	0.4230	0.0058	7
4) Nonsee Vitaya school	0.3519	0.3367	0.3745	0.3621	0.3563	0.0160	4
5) Singharatpitayakom school	0.4193	0.4004	0.4328	0.4169	0.4174	0.0133	6
6) Thonburi	0.4499	0.4451	0.4464	0.4411	0.4456	0.0036	8
7) Chokchai 4	0.3654	0.3609	0.3665	0.3615	0.3636	0.0028	5
8) Dindaeng	0.7386	0.7341	0.6180	0.6146	0.6763	0.0694	10
9) Meteorological Department	0.1890	0.1984	0.2448	0.2518	0.2210	0.0319	2
10) Ratburana	0.1382	0.1404	0.1831	0.1851	0.1617	0.0259	1

Table 3: Results of MCDM Method on Air Pollution Data in 2000.

`Monitoring Station	L1		L2		Mean	SD	Rank
	W1	W2	W1	W2			
1) Ramkhamheang University	0.4031	0.4041	0.4141	0.4150	0.4091	0.0064	9
2) National Housing Authority	0.2499	0.2470	0.2951	0.2930	0.2713	0.0264	4
3) Huai Khwang	0.3090	0.3190	0.3038	0.3126	0.3111	0.0064	7
4) Nonsee Vitaya school	0.2067	0.2305	0.2485	0.2660	0.2379	0.0254	2
5) Singharatpitayakom school	0.3392	0.3315	0.3600	0.3558	0.3466	0.0135	8
6) Thonburi	0.2907	0.2958	0.2909	0.2948	0.2930	0.0026	5
7) Chokchai 4	0.3010	0.3143	0.2929	0.3045	0.3032	0.0089	6
8) Dindaeng	0.7350	0.7544	0.6410	0.6489	0.6948	0.0582	10
9) Meteorological Department	0.1420	0.1320	0.1599	0.1548	0.1472	0.0126	1
10) Ratburana	0.2370	0.2243	0.3049	0.2983	0.2661	0.0414	3

Table 4: Results of MCDM Method on Air Pollution Data in 2001.

Monitoring Station	L1		L2		Mean	SD	Rank
	W1	W2	W1	W2			
1) Ramkhamheang University	0.4144	0.3669	0.4595	0.4356	0.4191	0.0394	8
2) National Housing Authority	0.3074	0.3006	0.3512	0.3411	0.3250	0.0248	7
3) Huai Khwang	0.2739	0.2891	0.2730	0.2850	0.2803	0.0081	6
4) Nonsee Vitaya school	0.1967	0.2082	0.2155	0.2226	0.2107	0.0111	3
5) Singharatpitayakom school	0.4501	0.4371	0.4554	0.4414	0.4460	0.0083	9
6) Thonburi	0.1853	0.2000	0.1865	0.1991	0.1927	0.0079	2
7) Chokchai 4	0.2237	0.2342	0.2238	0.2331	0.2287	0.0057	4
8) Dindaeng	0.5948	0.6508	0.5424	0.5680	0.5890	0.0464	10
9) Meteorological Department	0.1684	0.1544	0.1954	0.1852	0.1759	0.0181	1
10) Ratburana	0.2131	0.1946	0.2675	0.2539	0.2323	0.0341	5

From Tables 1-4, we observe that most often (1998, 2000, 2001) first rank is Meteorological Department station which means this station is expected to be good in terms of air pollution. On the other hand, Dindaeng station performed poorly. We selected these two stations to represent their performances graphically in Figures 2 – 3. These figures also depict their ranks for each season separately, rainy, summer and winter, along with the overall ranks. Details of seasonal analyses appear in the Technical Report. Returning to the air pollution data sets, we have applied Modifications I and II using various value of k to see ranks afresh. These are reported in Tables 5 – 12. The values of Spearman's rank correlation of two sets of ranks between MCDM method and Modifications I and II are shown in Table 13. Tables 14-15 show these values for Modifications I and II within themselves for different values of k. The robustness of the ranks is obvious in view of the large values of Spearman's rank correlation uniformly in all cases.

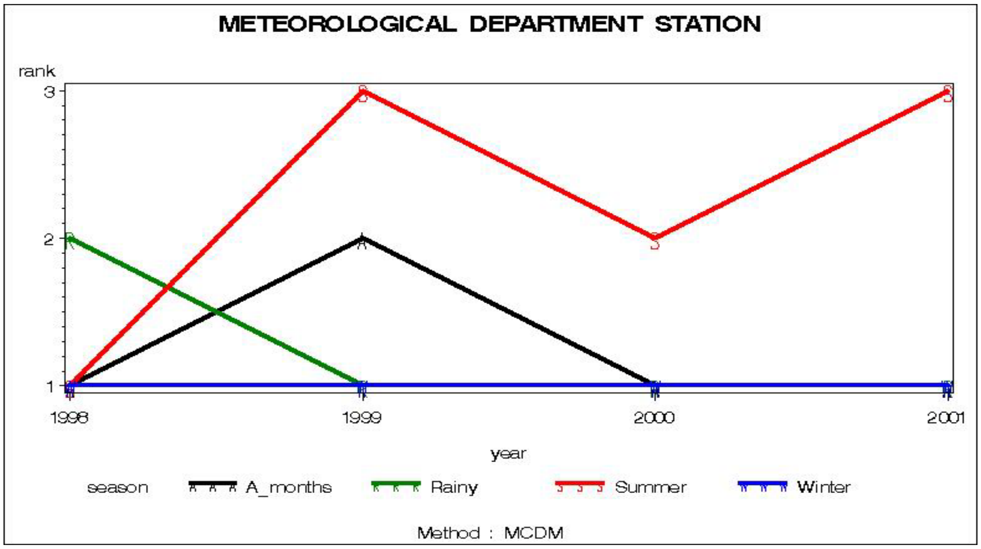

Figure 2 : Order of rank of Meteorological Department station for 1998-2001.

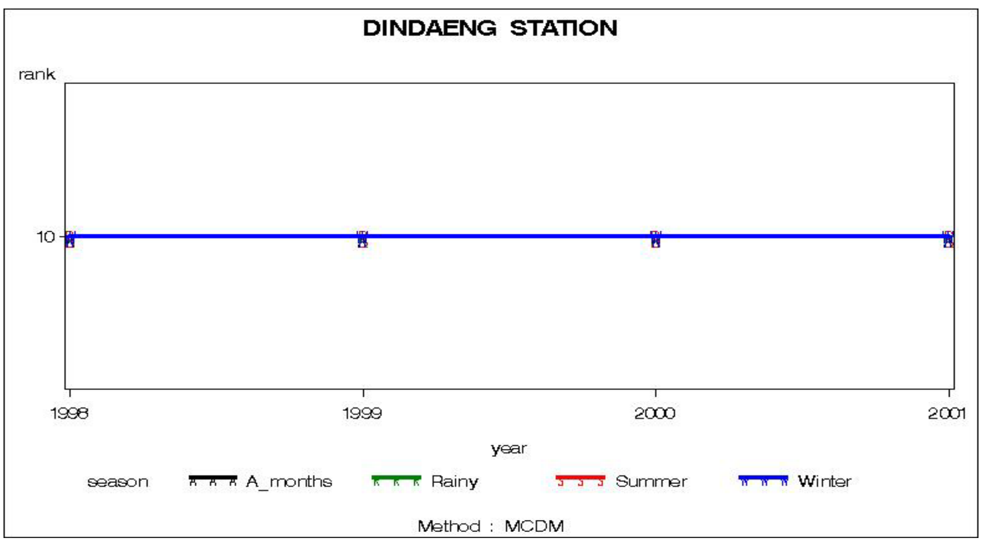

Figure 3 : Order of rank of Dindaeng station for 1998-2001.

Table 5: Results of Modification I for 1998.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.4108	10	0.2828	10	0.2065	10	0.1558	10	0.1198	10
2) National Housing Authority	0.1671	3	0.0805	3	0.0405	4	0.0208	4	0.0108	4
3) Huai Khwang	0.2304	6	0.1188	5	0.0627	5	0.0335	5	0.0180	5
4) Nonsee Vitaya School	0.1730	4	0.0819	4	0.0404	3	0.0205	3	0.0105	3
5) Singharatpitayakom School	0.3344	9	0.2449	9	0.1882	9	0.1472	9	0.1158	9
6) Thonburi	0.2561	7	0.1523	7	0.0944	7	0.0594	7	0.0376	7
7) Chokchai 4	0.2274	5	0.1261	6	0.0726	6	0.0425	6	0.0251	6
8) Dindaeng	0.2742	8	0.1642	8	0.1005	8	0.0623	8	0.0389	8
9) Meteorological Department	0.0345	1	0.0073	1	0.0016	1	0.0003	1	0.0001	1
10) Ratburana	0.1039	2	0.0389	2	0.0146	2	0.0055	2	0.0021	2

Table 6: Results of Modification II for 1998.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.6604	8	0.4202	7	0.2824	8	0.197	9	0.1413	9
2) National Housing Authority	0.5505	3	0.3467	4	0.226	4	0.1505	4	0.1017	5
3) Huai Khwang	0.6569	7	0.4246	8	0.2788	7	0.185	6	0.1239	6
4) Nonsee Vitaya School	0.5537	4	0.3426	3	0.2212	3	0.1471	3	0.1	3
5) Singharatpitayakom School	0.6155	5	0.3996	6	0.2703	6	0.1879	7	0.1334	8
6) Thonburi	0.6673	9	0.4347	9	0.2858	9	0.1887	8	0.125	7
7) Chokchai 4	0.6257	6	0.3934	5	0.2495	5	0.1589	5	0.1015	4
8) Dindaeng	0.8476	10	0.6385	10	0.4953	10	0.3902	10	0.3108	10
9) Meteorological Department	0.3119	1	0.1526	1	0.0773	1	0.04	1	0.0211	1
10) Ratburana	0.4187	2	0.2455	2	0.148	2	0.0905	2	0.0558	2

Table 7: Results of Modification I for 1999.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.2396	10	0.1050	8	0.0466	5	0.0208	4	0.0094	4
2) National Housing Authority	0.1482	2	0.0649	2	0.0291	2	0.0132	2	0.0060	2
3) Huai Khwang	0.2067	7	0.1002	7	0.0495	7	0.0247	7	0.0124	6
4) Nonsee Vitaya School	0.1520	3	0.0688	3	0.0321	3	0.0152	3	0.0073	3
5) Singharatpitayakom School	0.1927	6	0.0919	4	0.0453	4	0.0228	5	0.0116	5
6) Thonburi	0.2276	8	0.1179	9	0.0628	9	0.0340	9	0.0186	9
7) Chokchai 4	0.1921	5	0.0931	5	0.0468	6	0.0241	6	0.0125	7
8) Dindaeng	0.2342	9	0.1261	10	0.0688	10	0.0379	10	0.0210	10
9) Meteorological Department	0.1757	4	0.0966	6	0.0544	8	0.0310	8	0.0177	8
10) Ratburana	0.0855	1	0.0275	1	0.0091	1	0.0030	1	0.0010	1

Table 8: Results of Modification II for 1999.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.6537	9	0.4397	9	0.3188	9	0.2423	9	0.1898	9
2) National Housing Authority	0.553	3	0.3578	3	0.2403	5	0.1653	5	0.1157	5
3) Huai Khwang	0.6285	7	0.4071	7	0.2701	7	0.182	6	0.1241	6
4) Nonsee Vitaya School	0.5676	4	0.3629	4	0.2373	3	0.1571	4	0.1051	4
5) Singharatpitayakom School	0.6138	6	0.4001	6	0.269	6	0.1842	7	0.1278	7
6) Thonburi	0.652	8	0.4273	8	0.285	8	0.1921	8	0.1305	8
7) Chokchai 4	0.5992	5	0.3752	5	0.2383	4	0.1524	3	0.0981	3
8) Dindaeng	0.7974	10	0.5967	10	0.4617	10	0.3632	10	0.2888	10
9) Meteorological Department	0.4888	2	0.3073	2	0.1947	2	0.1236	2	0.0786	2
10) Ratburana	0.4172	1	0.2363	1	0.1401	1	0.0849	1	0.0521	1

Table 9: Results of Modification I for 2000.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.3624	10	0.2161	10	0.1348	10	0.0865	10	0.0566	10
2) National Housing Authority	0.1502	2	0.0638	2	0.0278	2	0.0123	2	0.0055	2
3) Huai Khwang	0.2027	6	0.1032	6	0.0542	6	0.0289	6	0.0155	6
4) Nonsee Vitaya School	0.1989	4	0.1320	8	0.0908	8	0.0631	8	0.0439	8
5) Singharatpitayakom School	0.2100	8	0.1025	5	0.0508	5	0.0255	5	0.0129	5
6) Thonburi	0.2076	7	0.1095	7	0.0594	7	0.0326	7	0.0180	7
7) Chokchai 4	0.1998	5	0.0981	4	0.0491	4	0.0248	4	0.0126	4
8) Dindaeng	0.2719	9	0.1697	9	0.1094	9	0.0722	9	0.0485	9
9) Meteorological Department	0.1257	1	0.0512	1	0.0214	1	0.0090	1	0.0038	1
10) Ratburana	0.1550	3	0.0731	3	0.0357	3	0.0178	3	0.0089	3

Table 10: Results of Modification II for 2000.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.6788	9	0.4461	9	0.3146	9	0.2317	9	0.1754	9
2) National Housing Authority	0.5353	4	0.3331	3	0.2171	6	0.1455	6	0.0994	6
3) Huai Khwang	0.5949	7	0.3576	7	0.2165	5	0.1315	5	0.0801	5
4) Nonsee Vitaya School	0.5111	2	0.3072	2	0.1892	2	0.1189	2	0.0761	2
5) Singharatpitayakom School	0.6024	8	0.385	8	0.2568	8	0.1766	8	0.1244	8
6) Thonburi	0.5856	5	0.3508	6	0.2121	4	0.1289	4	0.0788	4
7) Chokchai 4	0.5863	6	0.3486	5	0.209	3	0.126	3	0.0762	3
8) Dindaeng	0.8251	10	0.6191	10	0.486	10	0.3925	10	0.3234	10
9) Meteorological Department	0.4527	1	0.2481	1	0.1374	1	0.0765	1	0.0427	1
10) Ratburana	0.5246	3	0.3367	4	0.2272	7	0.1578	7	0.1117	7

Table 11: Results of Modification I for 2001.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.1658	6	0.0724	5	0.0318	5	0.0140	4	0.0062	3
2) National Housing Authority	0.1616	5	0.0789	6	0.0416	7	0.0232	7	0.0133	7
3) Huai Khwang	0.2043	8	0.1088	8	0.0595	8	0.0329	8	0.0182	8
4) Nonsee Vitaya School	0.1218	3	0.0519	3	0.0250	3	0.0129	3	0.0069	5
5) Singharatpitayakom School	0.2624	10	0.1467	10	0.0837	10	0.0484	10	0.0282	9
6) Thonburi	0.1500	4	0.0674	4	0.0311	4	0.0145	5	0.0068	4
7) Chokchai 4	0.1676	7	0.0802	7	0.0397	6	0.0199	6	0.0100	6
8) Dindaeng	0.2331	9	0.1368	9	0.0808	9	0.0479	9	0.0284	10
9) Meteorological Department	0.1083	2	0.0390	2	0.0143	2	0.0053	2	0.0020	2
10) Ratburana	0.0946	1	0.0330	1	0.0119	1	0.0044	1	0.0016	1

Table 12 : Results of Modification II for 2001.

Monitoring Station	k = 1	Rank	k = 1.5	Rank	k = 2	Rank	k = 2.5	Rank	k = 3	Rank
1) Ramkhamheang University	0.6731	8	0.4809	9	0.3613	9	0.279	9	0.2194	9
2) National Housing Authority	0.5902	6	0.384	7	0.2571	7	0.1751	7	0.1206	7
3) Huai Khwang	0.5999	7	0.3667	6	0.2259	6	0.1398	6	0.0869	6
4) Nonsee Vitaya School	0.5169	2	0.3014	3	0.1784	3	0.1064	3	0.0636	3
5) Singharatpitayakom School	0.7084	9	0.4761	8	0.3255	8	0.2251	8	0.1571	8
6) Thonburi	0.5207	3	0.2983	2	0.173	2	0.1012	1	0.0597	1
7) Chokchai 4	0.5541	5	0.3263	5	0.194	4	0.116	4	0.0697	4
8) Dindaeng	0.7948	10	0.5889	10	0.4497	10	0.349	10	0.274	10
9) Meteorological Department	0.4939	1	0.2872	1	0.1706	1	0.1024	2	0.0619	2
10) Ratburana	0.521	4	0.3244	4	0.2062	5	0.1324	5	0.0855	5

Table 13: Spearman's Rank Correlations Between MCDM and Modifications I and II.

Year	Modification I					Modification II				
	k=1	k=1.5	k=2	k=2.5	k=3	k=1	k=1.5	k=2	k=2.5	k=3
1998	0.7818	0.7333	0.7455	0.7455	0.7455	0.9030	0.9758	0.9394	0.8909	0.8667
1999	0.9515	0.8545	0.6364	0.6000	0.5758	1.0000	1.0000	0.9636	0.9394	0.9394
2000	0.9030	0.6364	0.6364	0.6364	0.6364	1.0000	0.9758	0.7939	0.7939	0.7939
2001	0.7333	0.7212	0.7576	0.6848	0.6182	0.9636	0.9758	0.9879	0.9758	0.9758

Table 14: Spearman's Rank Correlations for Modification I for Different Values of k.

Year	k=1, k=1.5	k=1, k=2	k=1, k=2.5	k=1, k=3	k=1.5, k=2	k=1.5, k=2.5	k=1.5, k=3	k=2, k=2.5	k=2, k=3	k=2.5, k=3
1998	0.9879	0.9758	0.9758	0.9758	0.9879	0.9879	0.9879	1.0000	1.0000	1.0000
1999	0.9152	0.7091	0.6606	0.6364	0.9152	0.8667	0.8424	0.9879	0.9758	0.9879
2000	0.8424	0.8424	0.8424	0.8424	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2001	0.9879	0.9636	0.9394	0.8788	0.9879	0.9758	0.9212	0.9879	0.9394	0.9515

Table 15: Spearman's Rank Correlations for Modification II for Different Values of k.

Year	k=1, k=1.5	k=1, k=2	k=1, k=2.5	k=1, k=3	k=1.5, k=2	k=1.5, k=2.5	k=1.5, k=3	k=2, k=2.5	k=2, k=3	k=2.5, k=3
1998	0.9636	0.9758	0.9394	0.8606	0.9879	0.9394	0.8909	0.9758	0.9273	0.9758
1999	1.0000	0.9636	0.9394	0.9394	0.9636	0.9394	0.9394	0.9758	0.9758	1.0000
2000	0.9758	0.7939	0.7939	0.7939	0.8182	0.8182	0.8182	1.0000	1.0000	1.0000
2001	0.9636	0.9515	0.9273	0.9273	0.9879	0.9758	0.9758	0.9879	0.9879	1.0000

The rest of this section is devoted to a discussion of the Electre Method described in Section 2.3. For every year (1998 – 2001), we begin with the data matrix X, and follow steps 1-5 to eventually obtain the matrix **E**. The four **E**-matrices are shown below in Table 16.

Table 16: E-matrices.

Station	1	2	3	4	5	6	7	8	9	10
1	0	1	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	0	0	1	1
3	1	1	0	1	1	0	0	0	1	1
4	1	1	0	0	0	0	0	0	1	1
5	1	1	0	1	0	0	0	0	1	1
6	1	1	1	1	1	0	1	0	1	1
7	1	1	1	1	1	0	0	0	1	1
8	1	1	1	1	1	1	1	0	1	1
9	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	1	0

Year 1998

Station	1	2	3	4	5	6	7	8	9	10
1	0	1	0	1	0	0	0	0	1	1
2	0	0	0	1	0	0	0	0	1	1
3	1	1	0	1	1	1	0	0	1	1
4	0	0	0	0	0	0	1	0	0	1
5	1	1	0	1	0	0	0	0	1	1
6	1	1	0	1	1	0	1	0	1	1
7	1	1	1	0	1	0	0	0	1	1
8	1	1	1	1	1	1	1	0	1	1
9	0	0	0	1	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0

Year 1999

Station	1	2	3	4	5	6	7	8	9	10
1	0	1	0	1	0	0	0	0	1	1
2	0	0	0	1	0	0	0	0	1	1
3	1	1	0	1	1	1	1	0	1	1
4	0	0	0	0	0	0	0	0	0	0
5	1	1	0	1	0	0	0	0	1	1
6	1	1	0	1	1	0	1	0	1	1
7	1	1	0	1	1	0	0	0	1	1
8	1	1	1	1	1	1	1	0	1	1
9	0	0	0	1	0	0	0	0	0	1
10	0	0	0	1	0	0	0	0	0	0

Year 2000

Station	1	2	3	4	5	6	7	8	9	10
1	0	1	0	1	0	0	0	0	1	1
2	0	0	0	1	0	0	0	0	1	1
3	1	1	0	1	0	1	1	0	1	1
4	0	0	0	0	0	0	0	0	0	1
5	1	1	1	1	0	1	1	0	1	1
6	1	1	0	1	0	0	0	0	1	1
7	1	1	0	1	0	1	0	0	1	1
8	1	1	1	1	1	1	1	0	1	1
9	0	0	0	1	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0

Year 2001

From the above table, we can conclude that in 1998, Meteorological Department station is the best and Ratburana station is the second. For 1999, the best station is Ratburana and the second best stations are Meteorological Department and Nonsee Vitaya school. In 2000, Nonsee Vitaya school and Ratburana are the best station and the second best station, respectively. Finally, in 2001, the best station is Ratburana and the second best station is Nonsee Vitaya School. In addition, the worst station is Dindeang for every year.

4. CONCLUSION

This paper presents a statistical study of the four air pollutants in the four-year period (1998-2001) from ten monitoring stations in Bangkok, Thailand using MCDM method. MCDM method is used to integrate the various columns of a data matrix so that each row is endowed with a single overall index, summarizing the different component indices over columns, thus making a ranking of the rows and hence their comparison feasible. Some modifications of MCDM are also used to rank the stations.

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